NCTU - Yau mini-course on string theory

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Construction of superstring field theory

The rules of the game

Step 1   Construct a kinetic term.

the equation of motion
$\longrightarrow$ the physical state condition

a gauge symmetry
$\longrightarrow$ the equivalence relation

Step 2   Include interaction terms.

invariance under a nonlinearly
extended gauge transformation

§1. Open bosonic string field theory

the physical state condition $Q\Psi = 0$

the equivalence relation $\Psi \sim \Psi + Q\Lambda$

Step 1

$$S = -\frac{1}{2}\langle \Psi, Q\Psi \rangle$$

$\delta S = 0$ under $\delta\Psi = Q\Lambda$

$$\langle A, B \rangle = (-1)^{AB}\langle B, A \rangle$$
$$Q^2 = 0$$
$$\langle A, QB \rangle = -(-1)^{A}\langle QA, B \rangle$$

Step 2

$$S = -\frac{1}{2}\langle \Psi, Q\Psi \rangle - \frac{g}{3}\langle \Psi, \Psi * \Psi \rangle$$

g : the open string coupling constant

$\delta S = 0$ under $\delta\Psi = Q\Lambda + g(\Psi * \Lambda - \Lambda * \Psi)$

$$Q(A * B) = QA * B + (-1)^{A} A * QB$$
$$(A * B) * C = A * (B * C)$$
$$\langle A, B * C \rangle = \langle A * B, C \rangle$$

associativity

nonassociative two-string product $V_2(A_1, A_2)$

$$S = -\frac{1}{2}\langle \Phi, Q\Phi \rangle - \frac{g}{3}\langle \Phi, V_2(\Phi, \Phi)\rangle$$

$$- \frac{g^2}{4}\langle \Phi, V_3(\Phi, \Phi, \Phi)\rangle + O(g^3)$$

three-string product
$V_3(A_1, A_2, A_3)$

$$(-1)^{V_2(A_1, A_2)} = (-1)^{A_1 + A_2}$$
$$(-1)^{V_3(A_1, A_2, A_3)} = (-1)^{A_1 + A_2 + A_3 + 1}$$

$$\langle A_1, V_2(A_2, A_3)\rangle = \langle V_2(A_1, A_2), A_3\rangle$$
$$\langle A_1, V_3(A_2, A_3, A_4)\rangle$$
$$= (-1)^{A_1}\langle V_3(A_1, A_2, A_3), A_4\rangle$$

$\delta S = O(g^3)$

under

$$\delta\Phi = Q\Lambda + g\,(V_2(\Phi, \Lambda) - V_2(\Lambda, \Phi))$$
$$+ g^2(V_3(\Phi, \Phi, \Lambda) - V_3(\Phi, \Lambda, \Phi)$$
$$+ V_3(\Lambda, \Phi, \Phi))$$
$$+ O(g^3)$$

if

$Q^2 = 0$,

$Q V_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{A_1} V_2(A_1, QA_2) = 0$,

$Q V_3(A_1, A_2, A_3)$

$- V_2(V_2(A_1, A_2), A_3) + V_2(A_1, V_2(A_2, A_3))$

$+ V_3(QA_1, A_2, A_3) + (-1)^{A_1} V_3(A_1, QA_2, A_3)$

$+ (-1)^{A_1 + A_2} V_3(A_1, A_2, QA_3) = 0$.

- These relations are extended to higher orders. $\longrightarrow$ A_∞ structure

- A_∞ structure

  $\updownarrow$ close relation

  the decomposition of the moduli space of Riemann surfaces

- A_∞ structure

  $\longrightarrow$ The Batalin-Vilkovisky quantization is straightforward.

gauge invariance as a spacetime theory

$\downarrow$ A_∞ structure

world-sheet picture

⑧ 1時間 35分
(10:20 ~ 11:55)

§2 Closed bosonic string field theory

Step 1

$$S = -\frac{1}{2} \langle \Psi, c_0^- Q \Psi \rangle$$

$$c_0^- = \frac{1}{2}(c_0 - \tilde{c}_0)$$

Ψ : ghost number 2
subject to the constraints

$$(b_0 - \tilde{b}_0)\Psi = 0$$

$$(L_0 - \tilde{L}_0)\Psi = 0$$

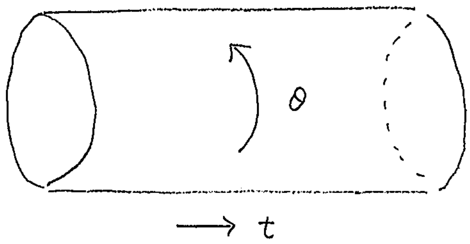
$\delta S = 0$ under $\delta \Psi = Q\Lambda$

$$(b_0 - \tilde{b}_0)\Lambda = 0$$

$$(L_0 - \tilde{L}_0)\Lambda = 0$$

the origin of the constraints

propagator surface



$$e^{-t(L_0+\tilde{L}_0)+i\theta(L_0-\tilde{L}_0)}$$

t, θ : moduli

integration over t $\longleftarrow$ propagator

$$\frac{b_0^+}{L_0^+} = \int_0^\infty dt \; b_0^+ e^{-tL_0^+}$$

$$L_0^+ = L_0 + \tilde{L}_0$$
$$b_0^+ = b_0 + \tilde{b}_0$$

integration over θ

$$B \equiv b_0^- \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta L_0^-}$$

$$L_0^- = L_0 - \tilde{L}_0$$
$$b_0^- = b_0 - \tilde{b}_0$$

$$\sim \delta(b_0^-)\,\delta(L_0^-)$$

implemented by the constraints

$$B = -i \int_0^{2\pi} \frac{d\theta}{2\pi} \int d\tilde{\theta}\ e^{i\theta L_0^- + i\tilde{\theta} b_0^-}$$

$\tilde{\theta}$: Grassmann-odd variable

(cf. extended BRST transformation
Witten, arXiv: 1209.5461)

$B c_0^- B = B \Rightarrow B c_0^-$: projector

The constraints can be characterized as
$$B c_0^- \Psi = \Psi .$$

① HRI 1.5 時間

Step 2 completed by Zwiebach in 1992

$$S = -\frac{1}{2} \langle \Psi, c_0^- Q \Psi \rangle - \sum_{n=1}^{\infty} \frac{g^n}{n+2} \langle \Psi, c_0^- V_{n+1} (\underbrace{\Psi, \Psi, \cdots, \Psi}_{n+1}) \rangle$$

the relations among the multi-string products for the closed string
$\longrightarrow$ L_∞ structure

② NCTU
(約 1.5 時間)

§3 Open superstring

bosonic string

$\downarrow$ spacetime bosons only

superstring

bosons and fermions

conformal field theory

$\downarrow$

superconformal field theory

$T_F(z)$, $\tilde{T}_F(\bar{z})$: supercurrents

$$T(z)\,T_F(0) \sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0)$$

$$T_F(z)\,T_F(0) \sim \frac{2c}{3z^3} + \frac{2}{z}T(0)$$

the superconformal ghost sector
the $\beta\gamma$ ghosts

β, γ : commuting fields
weights

$$\beta : \left(\frac{3}{2}, 0\right), \quad \gamma : \left(-\frac{1}{2}, 0\right)$$

the operator products

$$\beta(z_1)\,\gamma(z_2) \sim -\frac{1}{z_1 - z_2}$$

$$\gamma(z_1)\,\beta(z_2) \sim \frac{1}{z_1 - z_2}$$

two possible periodicity conditions
in the cylinder coordinate

the Neveu - Schwarz (NS) sector

$$T_F(w+2\pi) = -T_F(w)$$
$$\beta(w+2\pi) = -\beta(w)$$
$$\gamma(w+2\pi) = -\gamma(w)$$

the Ramond sector

$$T_F(w+2\pi) = T_F(w)$$
$$\beta(w+2\pi) = \beta(w)$$
$$\gamma(w+2\pi) = \gamma(w)$$

We use the doubling trick
for the open superstring.

the NS sector $\rightarrow$ spacetime bosons
the Ramond sector $\rightarrow$ spacetime fermions

§4 Open superstring field theory: the NS sector

mode expansions

$$\beta(z) = \sum_r \frac{\beta_r}{z^{r+\frac{3}{2}}}, \qquad \gamma(z) = \sum_r \frac{\gamma_r}{z^{r-\frac{1}{2}}}$$

$r = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots$

$$[\gamma_r, \beta_s] = \delta_{r,-s}$$

$$\beta_r |0\rangle_{NS} = 0 \quad \text{for} \quad r \geqq \frac{1}{2}$$

$$\gamma_r |0\rangle_{NS} = 0 \quad \text{for} \quad r \geqq \frac{1}{2}$$

this choice $\longrightarrow$ -1 picture

The operator corresponding to $|0\rangle_{NS}$ is denoted by $\delta(\gamma(0))$.

ghost number $\sim$ bosonic moduli
picture number $\sim$ fermionic moduli

Step 1

$$S = -\frac{1}{2} \langle \Psi, Q\Psi \rangle$$

Ψ ; ghost number 1
picture number -1
Q ; the BRST operator
in the open superstring

$\delta S = 0$ under $\delta\Psi = Q\Lambda$

Step 2

$$S = -\frac{1}{2} \langle \Psi, Q\Psi \rangle - \frac{g}{3} \langle \Psi, \Psi * \Psi \rangle \ ?$$

$\uparrow$

total picture -3

The total picture for a BPZ
inner product has to be -2

a local insertion of
a picture-changing operator
$\downarrow$
singularity

~~Step 2~~

The Berkovits formulation

The superconformal ghost sector can also be described by $\underbrace{\xi(z),\ \eta(z)}_{\text{anticommuting fields}}$, and $\phi(z)$.

linear dilaton CFT

	weight	picture
$\xi(z)$	0	1
$\eta(z)$	1	-1
$e^{\ell\phi}(z)$	$-\frac{1}{2}\ell^2-\ell$	ℓ

$\beta(z)$, $\gamma(z)$: the small Hilbert space

$\xi(z)$, $\eta(z)$, $\phi(z)$: the large Hilbert space

$A \in$ the small Hilbert space

$\Longleftrightarrow \quad \eta A = 0$

η : the zero mode of $\eta(z)$

BPZ inner products

$$\langle\langle A, B \rangle\rangle = \langle \xi_0 A, B \rangle$$

small   large

ξ_0 : the zero mode of $\xi(z)$

The total picture has to be -1.

Step 1

$$S = -\frac{1}{2} \langle \Phi, Q\eta\Phi \rangle$$

$\Phi \in$ the large Hilbert space
ghost number 0
picture number 0

$\delta S = 0$ under $\delta\Phi = Q\Lambda + \eta\Omega$

$$\eta^2 = 0$$
$$\{Q, \eta\} = 0$$
$$\langle \eta A, B \rangle = -(-1)^A \langle A, \eta B \rangle$$

$\exists\ \xi$ in the large Hilber space
satisfying
$$\{\eta, \xi\} = 1$$
(For example, $\xi = \xi_0$.)

$$\Phi = \eta\xi\Phi + \xi\eta\Phi$$
$$\downarrow \quad \delta\Phi = \eta\Omega$$
$\Phi = \xi\Psi$ with $\Psi = \eta\Phi$
in the small Hilbert space

$$Q\eta\Phi = Q\eta\xi\Psi = Q\{\eta, \xi\}\Psi = Q\Psi = 0$$

⑨ 1時間45分
(13:05 ~ 14:50)
② HRI 約1.5時間

Algebraic relations in the large Hilbert space

$$\langle B, A \rangle = (-1)^{AB} \langle A, B \rangle ,$$
$$Q^2 = 0, \quad \eta^2 = 0, \quad \{ Q, \eta \} = 0$$
$$\langle QA, B \rangle = -(-1)^{A} \langle A, QB \rangle ,$$
$$\langle \eta A, B \rangle = -(-1)^{A} \langle A, \eta B \rangle ,$$
$$\langle A, B * C \rangle = \langle A * B, C \rangle ,$$
$$(A * B) * C = A * (B * C) ,$$
$$Q(A * B) = QA * B + (-1)^{A} A * QB ,$$
$$\eta(A * B) = \eta A * B + (-1)^{A} A * \eta B .$$

Step 2

($A * B \to AB$ in what follows)

$O(g^2)$

$$S = -\frac{1}{2}\langle \Phi, Q\eta\Phi\rangle - \frac{g}{6}\langle \Phi, Q[\Phi, \eta\Phi]\rangle$$

$$- \frac{g^2}{24}\langle \Phi, Q[\Phi, [\Phi, \eta\Phi]]\rangle + O(g^3)$$

$$\delta\Phi = Q\Lambda + \eta\Omega - \frac{g}{2}[\Phi, Q\Lambda] + \frac{g}{2}[\Phi, \eta\Omega]$$

$$+ \frac{g^2}{12}[\Phi, [\Phi, Q\Lambda]] + \frac{g^2}{12}[\Phi, [\Phi, \eta\Omega]]$$

$$+ O(g^3)$$

the WZW-like form

Berkovits, hep-th/9503099

$$S = \frac{1}{2}\langle e^{-\Phi}Qe^{\Phi}, e^{-\Phi}\eta e^{\Phi}\rangle$$

$$-\frac{1}{2}\int_0^1 dt \langle e^{-\Phi(t)}\partial_t e^{\Phi(t)},$$

$$\{e^{-\Phi(t)}Qe^{\Phi(t)}, e^{-\Phi(t)}\eta e^{\Phi(t)}\}\rangle$$

$$\Phi(1) = \Phi$$

$$\Phi(0) = 0$$

$$g = 1$$

Note: no A_∞ structure

Another form

$$S = -\int_0^1 dt \, \langle A_t(t), \, Q A_\eta(t) \rangle$$

$$A_\eta(t) = (\eta e^{\Phi(t)}) e^{-\Phi(t)}$$

$$A_t(t) = (\partial_t e^{\Phi(t)}) e^{-\Phi(t)}$$

t-dependence : topological

the gauge transformations

$$A_\delta = Q\Lambda + D_\eta \Omega$$

$$A_\delta = (\delta e^{\Phi}) e^{-\Phi}$$

$$D_\eta A = \eta A - A_\eta A + (-1)^A A A_\eta$$

with $A_\eta = (\eta e^{\Phi}) e^{-\Phi}$

"covariant derivative"

$$\boxed{\begin{aligned} &\eta A_\eta(t) - A_\eta(t) A_\eta(t) = 0 \\ &\partial_t A_\eta(t) = \eta A_t(t) - A_\eta(t) A_t(t) + A_t(t) A_\eta(t) \end{aligned}}$$

$\rightarrow$ gauge invariance,
topological t-dependence

The partial gauge fixing $\Phi \to \xi\Phi$
can be extended to the interacting theory.
$\longrightarrow$ a regular formulation
in the small Hilbert space

Iimori, Noumi, Okawa and Torii,
arXiv: 1312.1677

$\chi = \{ Q, \xi \}$: a line integral
of the picture-changing
operator
$\longrightarrow$ no singularities

an action with an A_∞ structure
using ξ as a new ingredient
Erler, Konopka and Sachs,
(in Munich)
arXiv: 1312.2948

The Munich construction

$$V_2(A_1, A_2)$$
$$= \frac{1}{3}\left[\chi(A_1 A_2) + (\chi A_1) A_2 + A_1(\chi A_2) \right]$$

non associative

$\longrightarrow$ We need V_3, but the construction is complicated.

Idea

Consider the following field redefinition of the free theory:

$$S = -\frac{1}{2} \langle\langle \tilde{\Phi}, Q\tilde{\Phi} \rangle\rangle,$$

$$\tilde{\Phi} = \Phi + g\,\tilde{V}_2(\Phi, \Phi) + O(g^2).$$

If $V_2(A_1, A_2)$ satisfies

$$(-1)^{\tilde{V}_2(A_1, A_2)} = (-1)^{A_1 + A_2 + 1},$$
$$\langle\langle A_1, \tilde{V}_2(A_2, A_3) \rangle\rangle = (-1)^{A_1} \langle\langle \tilde{V}_2(A_1, A_2), A_3 \rangle\rangle,$$

the action can be written as

$$S = -\frac{1}{2} \langle\langle \tilde{\Phi}, Q\tilde{\Phi} \rangle\rangle$$

$$= -\frac{1}{2} \langle\langle \Phi, Q\Phi \rangle\rangle - \frac{g}{3} \langle\langle \Phi, V_2(\Phi, \Phi) \rangle\rangle + O(g^2),$$

where

$$V_2(A_1, A_2)$$
$$= Q\tilde{V}_2(A_1, A_2) + \tilde{V}_2(QA_1, A_2) + (-1)^{A_1}\tilde{V}_2(A_1, QA_2).$$

We can show that the resulting two-string product $V_2(A_1, A_2)$ satisfies

$$(-1)^{V_2(A_1, A_2)} = (-1)^{A_1 + A_2},$$

$$\langle\!\langle A_1, V_2(A_2, A_3) \rangle\!\rangle = \langle\!\langle V_2(A_1, A_2), A_3 \rangle\!\rangle,$$

and one of the A_∞ relations

$$Q V_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{A_1} V_2(A, QA_2) = 0.$$

This way we can generate a set of multi-string products satisfying the A_∞ relations by field redefinition, but this is just a complicated way of describing the free theory.

The two-string product

$$V_2(A_1, A_2) = \frac{1}{3}\Big[X(A_1 A_2) + (X A_1) A_2 + A_1 (X A_2) \Big]$$

can be written as

$$V_2(A_1, A_2) = Q \widetilde{V}_2(A_1, A_2) + \widetilde{V}_2(QA_1, A_2) + (-1)^{A_1} \widetilde{V}_2(A_1, QA_2)$$

with

$$\widetilde{V}_2(A_1, A_2) = \frac{1}{3}\Big[\xi(A_1 A_2) + (\xi A_1) A_2 + (-1)^{A_1} A_1 (\xi A_2) \Big].$$

This choice of $V_2(A_1, A_2)$ satisfying the A_∞ relation can be formally understood as being generated from the following field redefinition:

$$\tilde{\Phi} = \Phi + \frac{g}{3}\left[\xi(\Phi\Phi) + (\xi\Phi)\Phi - \Phi(\xi\Phi)\right]$$

$$+ O(g^2).$$

However, this is actually an "illegal" field redefinition because

$$\eta\tilde{\Phi} \neq 0.$$

This is why the resulting theory can be interacting.

While $\eta \tilde{\Phi} \neq 0$, the multi-string products have to be in the small Hilbert space.

We can show

$$\eta \tilde{\Phi} - \tilde{\Phi}^2 = 0$$

$$\downarrow$$

$V_n \in$ the small Hilbert space

the same relation as

$$\eta A_\eta(t) - A_\eta(t)^2 = 0$$

In fact, the Munich action can be brought to the WZW-like form where $A_\eta(t)$ and $A_t(t)$ are parameterized in a complicated way.

	The Berkovits formulation	The Munich construction
action	beautifully written in the WZW-like form	not written in a closed form
A_∞	No	Yes

equivalence

Erler, Okawa and Takezaki, arXiv: 1505.01659

Erler, arXiv: 1505.02069

1510.00364

③ NCTU

(約2時間)

§5 Open superstring field theory : the Ramond sector

mode expansion

$$\beta(z) = \sum_r \frac{\beta_r}{z^{r+\frac{3}{2}}} , \qquad \gamma(z) = \sum_r \frac{\gamma_r}{z^{r-\frac{1}{2}}}$$

$r = 0, \pm 1, \pm 2, \cdots$

$$[\gamma_r, \beta_s] = \delta_{r,-s}$$

$\beta_r |0\rangle_R = 0$ for $r \geq 0$

$\gamma_r |0\rangle_R = 0$ for $r \geq 1$

this choice $\rightarrow -\frac{1}{2}$ picture

③ HRI 1.5時間

Consider a state A of picture number $-1/2$ in the small Hilbert space.
We say that A is in the restricted space when A satisfies

$$X Y A = A ,$$

where

$$X = G_0 \, \delta(\beta_0) + b_0 \, \delta'(\beta_0) ,$$
$$Y = - c_0 \, \delta'(\gamma_0)$$

with G_0 being the zero mode of $T_F(z)$.

$XYX = X \Rightarrow XY$: projector

$[Q, X] = 0$

Step 1

$$S = -\frac{1}{2} \langle\langle \Psi , Y Q \Psi \rangle\rangle$$

$\Psi \in$ the small Hilbert space
ghost number 1
picture number $-1/2$
$XY\Psi = \Psi$

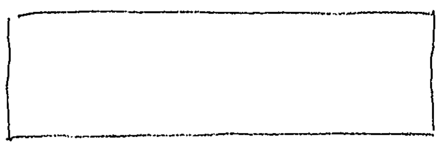
$\delta S = 0$ under $\delta \Psi = Q\lambda$

$\uparrow$

$XY\lambda = \lambda$

the origin of the restriction

propagator strip
for the Ramond sector



$$e^{-tL_0 + \zeta G_0}$$

$\longrightarrow t$ bosonic modulus t
$+$
fermionic modulus ζ

integration over ξ

$$X = \int d\xi \int d\tilde{\xi} \; e^{\xi G_0 - \tilde{\xi} \beta_0}$$

$\tilde{\xi}$: Grassmann-even variable
(not an ordinary number!)
Witten, arXiv: 1209.5461
Ohmori and Okawa,
arXiv: 1703.08214

$$= G_0 \, \delta(\beta_0) + b_0 \, \delta'(\beta_0)$$

$$X Y \Psi = \Psi$$

$$\updownarrow$$

$B c_0^- \Psi = \Psi$ for the closed string field ⅃

⑩ 1時間 25分
(9:05 ~ 10:30)

Step 2

Let us choose the free theory to be

$$S^{(0)} = -\frac{1}{2}\langle \Phi, Q\eta\Phi\rangle - \frac{1}{2}\langle\!\langle \Psi, YQ\Psi \rangle\!\rangle$$

$$\delta S^{(0)} = 0$$

under
$$\delta^{(0)}\Phi = Q\Lambda + \eta\Omega,$$
$$\delta^{(0)}\Psi = Q\lambda.$$

We want to write $X = \{Q, \Xi\}$.
Roughly speaking, $\Xi = \Theta(\beta_0)$.
See Erler, Okawa and Takezaki,
arXiv: 1602.02582
for a refined definition.

$$S = S^{(0)} + g\, S^{(1)} + g^2\, S^{(2)} + O(g^3)$$
$$\delta\Phi = \delta^{(0)}\Phi + g\,\delta^{(1)}\Phi + O(g^2)$$
$$\delta\Psi = \delta^{(0)}\Psi + g\,\delta^{(1)}\Psi + O(g^2)$$

$$S^{(1)} = -\frac{1}{6}\langle \Phi, Q[\Phi, \eta\Phi]\rangle - \langle \Phi, \Psi^2\rangle$$

$$S^{(2)} = -\frac{1}{24}\langle \Phi, Q[\Phi, [\Phi, \eta\Phi]]\rangle$$
$$-\frac{1}{2}\langle \Phi, \{\Psi, \Xi\{\eta\Phi, \Psi\}\}\rangle$$

$$\delta^{(1)}\Phi = -\frac{1}{2}[\Phi, Q\Lambda] + \frac{1}{2}[\Phi, \eta\Omega] - \{\Psi, \Xi\lambda\}$$

$$\delta^{(1)}\Psi = X\eta\{\Psi, \Lambda\} - X\eta\{\eta\Phi, \Xi\lambda\}$$

consistent with $XY\delta\Psi = \delta\Psi$

Complete action

Kunitomo and Okawa,
arXiv: 1508.00366

$$S = -\frac{1}{2} \langle\langle \Psi, Y Q \Psi \rangle\rangle - \int_0^1 dt \, \langle A_t(t), \, Q A_\eta(t) + (F(t)\Psi)^2 \rangle,$$

where

$$A_\eta(t) = (\eta \, e^{\Phi(t)}) \, e^{-\Phi(t)},$$

$$A_t(t) = (\partial_t \, e^{\Phi(t)}) \, e^{-\Phi(t)},$$

$$F(t)\Psi = \Psi + \Xi \{ A_\eta(t), \Psi \} + \Xi \{ A_\eta(t), \Xi \{ A_\eta(t), \Psi \} \} + \cdots$$

We can also use the Munich construction for the NS sector.
$\longrightarrow$ a complete action
with an A_∞ structure
Erler, Okawa and Takezaki,
arXiv: 1602.02582
Konopka and Sachs,
arXiv: 1602.02583

The approach to the Ramond sector by Sen

Sen, arXiv: 1508.05387

originally in closed superstring field theory

$\longrightarrow$ open superstring field theory

Step 1

$$S = \frac{1}{2} \langle\langle \widetilde{\Psi}, Q X_0 \widetilde{\Psi} \rangle\rangle - \langle\langle \widetilde{\Psi}, Q \Psi \rangle\rangle$$

Ψ : ghost number 1
picture number $-1/2$

$\widetilde{\Psi}$: ghost number 1
picture number $-3/2$

no constraints

$\beta_r |0\rangle_R = 0$ for $r \geqq 1$
$\gamma_r |0\rangle_R = 0$ for $r \geqq 0$

X_0 : the zero mode of
the picture-changing operator

$\Psi \rightarrow$ correct spectrum
$\widetilde{\Psi} \rightarrow$ extra free fields

Step 2

Use the interactions of the complete actions with $\Xi \to \xi_0$.

The equations of motion

$$Q\,\widetilde{\Psi} = \mathcal{J}(\Phi, \Psi) \qquad \Phi : \text{NS sector}$$

$$Q\,\Psi - Q X_0 \widetilde{\Psi} = 0$$

$$\Longrightarrow \quad Q\,\Psi = X_0\, \mathcal{J}(\Phi, \Psi)$$

fluctuations of $\widetilde{\Psi}$: free fields

Everything can be described by $\xi(z)$, $\eta(z)$, and $\phi(z)$.

closed superstring field theories

§6 Future directions

Improve open superstring field theory.

key ingredients

- the supermoduli
- A_∞ structure
- the large Hilbert space

scattering amplitudes
Iimori, Noumi, Okawa and Torii,
arXiv: 1312.1677
Kunitomo, Okawa, Sukeno and Takezaki,
arXiv: 1612.00777
the large Hilbert space
$\longrightarrow$ the supermoduli

the $\beta\gamma$ ghosts
without using the large Hilbert space
Ohmori and Okawa,
arXiv: 1703.08214

the relation to the approach by Sen
Okawa and Sakaguchi, to appear ⌟

⑪ 1時間15分 (10:40～11:55)
④ NCTU (約1時間45分)
④ HRI 約1.5時間